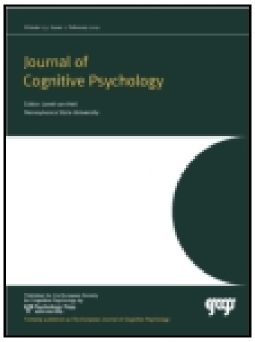

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Cognitive change in mental models with experience in the domain of organic chemistry

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We examined cognitive change in students' mental models, and consequently their problem-solving strategies, as a result of instruction in the domain of organic chemistry. Three groups of students received organic chemistry instruction that emphasised either imagistic strategies, analytic problem-solving strategies, or their combination. Before instruction, students' solution strategies were largely imagistic. After instruction, imagistic strategies comprised a minority of the strategies reported, indicating a switch from analogue mental models to more abstract representations. This switch was moderated by instruction and ability such that students who received analytic instruction used more analytic strategies after instruction and students with higher spatial ability used more imagistic strategies after instruction. Problem-solving success was associated with using a greater range of strategies. These results are consistent with research in other domains suggesting that imagistic mental models are associated with novelty, and as students gain more experience in a domain, they adopt domain-specific heuristics and rules when possible.

Keywords: Chemistry problem solving; Imagistic strategies; Analytic strategies; Instruction; Spatial ability.

There have been two different senses of mental model in the psychological literature. In one sense, often adopted in studies of reasoning (Johnson-Laird, 1983) and text comprehension (Van Dijk & Kintsch, 1983), a mental model (or situation model) is a representation of the situation or possible range of situations described by a set of premises or a more integrated text. In this case the input to the reasoning process is typically verbal. In another sense, a mental model is a characterisation of the knowledge and cognitive processes that allow humans to understand and predict the behaviour of physical systems such as mechanical, electronic, and biological systems (Gentner & Stevens, 1983), and the input to the reasoning process is often a picture or diagram. This sense of mental model does not make any strong predictions about the format of the knowledge representations or inferences involved. Rather, it can involve different types of representations at different levels of abstraction from the physical situation. This paper is in the second tradition and examines cognitive change in students' mental models, and consequently their problem-solving strategies, as they gain knowledge in the domain of organic chemistry.

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Different modes of reasoning about physical systems suggest that problem solvers represent such systems with mental models that vary from more imagistic to more abstract and analytic. For example, in mechanical reasoning, simulation-based reasoning involves forming a mental image of a mechanical system and using analogue imagery processes (e.g., mental rotation) to infer the behaviour of the system. We will refer to this process as running an imagistic mental model or using an imagistic strategy. Analytic reasoning, on the other hand, can involve decomposing the mechanical system into subsystems or applying rule-based strategies to infer how the system will behave. Take, for example, a simple mechanical reasoning problem with a picture or diagram representing interconnected gears. The direction of motion of one gear is indicated and the goal is to infer the direction of another gear in the chain. Solving this problem via simulation-based reasoning involves forming an imagistic mental model of the gears and mentally simulating their motions using visual and perhaps motor imagery. Solving this problem via analytic reasoning involves identifying the number of interconnected gears and applying the rule that two interlocking gears move in opposite directions (Schwartz & Black, 1996).

Another type of analytic strategy, task decomposition, is often used when reasoning about complex physical systems. For example, using task decomposition, participants mentally simulate the behaviour of complex mechanical systems piecemeal rather than holistically (Hegarty, 1992, 2004). The choice of internal representation and reasoning strategy can depend on how the problem is presented. People are more likely to use imagistic simulation when viewing a realistic picture of a mechanical system; conversely, when shown a more abstract diagram, they are more likely to use analytic strategies (Schwartz, 1995). Diagrams typically abstract from the reality that they represent so that they highlight some spatial aspects of the referent but not others. So although diagrams are models of the situation, they are typically more abstract than images and abstraction may prompt people to use more analytic inference processes. The presence of a diagram also facilitates piecemeal strategies, as it provides an external representation that can be viewed piece by piece, and obviates the need to maintain an internal representation, such as a mental image, in working memory. Furthermore, reasoners can annotate the external diagram as they solve parts of a problem to further reduce the working memory load.

Running imagistic mental models has also been associated with novel problem solving. For example, when Schwartz and Black (1996) asked people to solve the gear problem described earlier, participants' gestures indicated that they initially mentally simulated the motion of the individual gears, but on the basis of these simulations, they discovered the simple rule that any two interlocking gears must move in opposite directions and switched to the rule-based strategy. However, they reverted to the mental simulation strategy when given a novel type of gear problem. Similarly, when studying the strategies adopted by students to solve mathematics word problems, Lowrie and Kay (2001) found that students are more likely to report imagistic thinking processes (such as imagining the situation described in the problem) when solving more difficult problems. They argued that imagistic strategies are used in novel problem-solving situations in which students do not have a readily available algorithm, but algorithms are used, when available, because they are less effortful.

Chemistry is an interesting domain in which to examine mental models of physical systems. The objects under study in chemistry, namely atoms and molecules, exist on such a small scale that we have no direct visual experience on which to base our mental models in this domain. In response, chemists have invented a variety of physical models and diagrams to represent molecules, raising questions regarding how these external representations influence learners' developing mental models and strategy use.

Like mechanical and mathematical reasoning, studies in chemistry have revealed a variety of reasoning strategies, ranging from imagistic simulations to rule-based reasoning, that reflect variation in learners' mental models. For example, Stieff (2007) examined how novices and experts in organic chemistry solved a problem in which they were shown two molecular diagrams and tasked with deciding if the two diagrams represented the same molecule or a mirror image pair. Judgements such as this are critical in organic chemistry, given that two molecules may be composed of the same atoms with different spatial configurations that result in very different physical and chemical properties. Novices used mental rotation to perform this task, but experts typically used a rule-based strategy when applicable. Specifically, experts were able determine that two diagrams were identical if either of the two diagrams was symmetrical around a central axis. Using this rule, experts' response times were significantly faster and independent of angular disparity. Novices did not use this rule spontaneously, despite exposure to the rule through routine instruction; however, when directly instructed to use the rule, novices switched to rule-based reasoning instead of imagery. Interestingly, both experts and novices fell back on the mental rotation strategy on items to which the rule could not be applied.

More generally, a switch from imagistic to rule-based mental models with increasing expertise is evident across a range of organic chemistry problems that involve predicting reaction products, translating between different molecular representations, and determining the mechanisms underlying chemical reactions (see examples in Figure 1). Verbal protocols reveal that both experts and novices use analytic strategies for problem solving on these tasks, but experts use them more frequently and consistently, whereas novices rely more on imagistic mental models (Stieff, 2011; Stieff & Raje, 2010). Stieff, Ryu, Dixon, and Hegarty (2012) demonstrated that the switch from imagistic to analytic strategies occurs naturally over the course of instruction. When we asked organic chemistry students report their strategies on chemistry problems immediately after they had covered the relevant content in their classes, students reported primarily imagistic strategies. However, when asked to report their strategies at the end of the semester, students reported more analytic strategies, including diagram manipulation and rule-based reasoning.

Research in the domains of mechanics, mathematics, and chemistry has therefore revealed that learners shift from more imagistic to more abstract mental models and reasoning processes with experience in a domain. Here we examine how this development is moderated by the strategies that are emphasised in instruction and by students' spatial abilities. Although changes in students' mental models result at least partially from spontaneous discovery of rules (e.g., Schwartz & Black, 1996), it is clear that a major factor in this cognitive change is that students are exposed to a variety of domain-specific models and strategies in the course of instruction. However, students do not always adopt strategies that they are taught (Stieff, 2007; Lowrie, 1996). Thus, it is unclear how students' thinking processes are influenced by the strategies emphasised by their instructor and by their abilities.

To clarify these issues, we studied changes in students' mental models and problem-solving strategies in three organic chemistry classes taught by the same instructor, but emphasising different strategies. In one semester the instructor emphasised physical models and imagistic simulations, in a second semester she emphasised applying analytic rules and heuristics, and in a third she gave equal emphasis to both types of strategies. We expected that students would switch from primarily imagistic to more analytic methods as they gained more domain knowledge. We predicted that this switch would be more evident for students who were exposed to analytic strategies in instruction. We also examined whether students' spatial abilities affected the switch from imagistic to analytic thinking. A shift to analytic thinking might be characterised as one of adopting less effortful strategies. Given that imagistic strategies should be less effortful for higher spatial ability students than for lower spatial ability students, we speculate that higher spatial students might be less likely to switch to analytic strategies, regardless of how they are taught.

Finally, we examined the use of different types of analytic strategies that varied in the extent to which they relied on internal representations (mental images) versus external representations (diagrams), imagistic versus analytic processes, and the amount of spatial information that was considered in solving the problems. Imagistic strategies were those that relied on construction of an internal visuospatial image of the molecule or situation given in the problem and involved analogue imagery processes to derive the answer. Diagrammatic strategies relied on external visuospatial representations and processes that involved modifying existing diagrams or re-representing the problem in a different diagrammatic format. Spatial analytic strategies involved rules and heuristics that operated on spatial information extracted from a diagram or verbal statement of a problem. Algorithmic strategies involved rules and heuristics that operated on nonspatial information extracted from a diagram or verbal statement (see examples in Table 1).

To assess strategies we used a method developed by mathematics educators (Lean & Clements, 1981). This involves first conducting protocol studies to discover the range of strategies used by students to solve a set of problems and then developing strategy-choice questionnaires that ask students first to solve a set of problems and then choose from a set of strategy descriptions the

TABLE 1
Examples of each type of strategy (Stieff et al., 2012)

Strategy type	*Sample strategy responses*
Spatial-imagistic	I tend to imagine the molecule in 3D and rotate it ‘‘in my head’’. I tend to imagine myself moving into the paper or around the molecule.
Spatial-diagrammatic	I tend to first draw a basic skeletal structure and then make changes as I go. I tend to redraw the molecule using a different chemical representation to help me think about it.
Spatial-analytic	I tend to assign R/S labels to each molecule.
Algorithmic	I just know that in stable molecules particular groups must be in a specific relationship. I tend to use a specific formula to calculate the number of stereoisomers.

method(s) that they used to solve each problem. Although there are limitations of self-report data of this type, pilot research for the study presented here indicated that strategy classifications based on verbal protocols and students’ self-reports were consistent.

METHOD

Participants

Students were recruited from three introductory organic chemistry courses taught at a research university by the same instructor in successive semesters. The sample consisted of 469 students, 136 students (53 male, 83 female) in the class that emphasised analytic problem solving, 158 students (52 male, 106 female) in the class that emphasised imagistic problem solving, and 185 students (71 male, 104 female) in the class that emphasised both types of strategies. The samples consisted of 70% or more of the students enrolled in each class and did not differ significantly in self-reported Scholastic Aptitude Test (SAT) scores or spatial abilities.

Materials

Measures of spatial ability. Students were tested on three measures of spatial ability, the Mental Rotation test (Vandenberg & Kuse, 1978), the Paper Folding test (Ekstrom, French, Harman, & Dermen, 1976), and a modified version of Guay’s Visualisation of Views test (Guay & McDaniel, 1976).

Measure of organic chemistry problem solving and strategy choice. The Organic Chemistry Problem Solving test consisted of 12 problems that assessed student understanding of spatial relationships relevant to organic molecules and organic transformations (two of the problems are shown in Figure 1). Each item required students (1) to identify spatial relationships between molecules or substituents (groups of atoms) within a molecule and (2) self-report one or more strategies used to solve each problem immediately after solving the problem. The measure of problem-solving performance was the total number of problems answered correctly. The Pearson correlation of this measure with course grade was .58 ($p < .001$), indicating that it was representative of student achievement in the class. Participants were asked to report the strategy (or strategies) they used to solve each item by selecting from a fixed list of applicable strategies, adapted from previous studies (Stieff, 2011; Stieff & Raje, 2010). They were allowed to report more than one strategy per problem and to write in their own strategy if none of the choices matched it.

Procedure

The spatial ability measures were administered online during the first week of each semester with the standard test instructions and time limits. At this time the students also completed a questionnaire in which they reported their SAT scores. The 12-item organic chemistry problem-solving test was administered during the first and final weeks of each semester.

In each semester, strategy training was delivered via three 1-hour workshops that were supplementary to normal curriculum activities and emphasised in lecture. Both the courses and workshops were delivered by the same female instructor who had 7 years of experience teaching

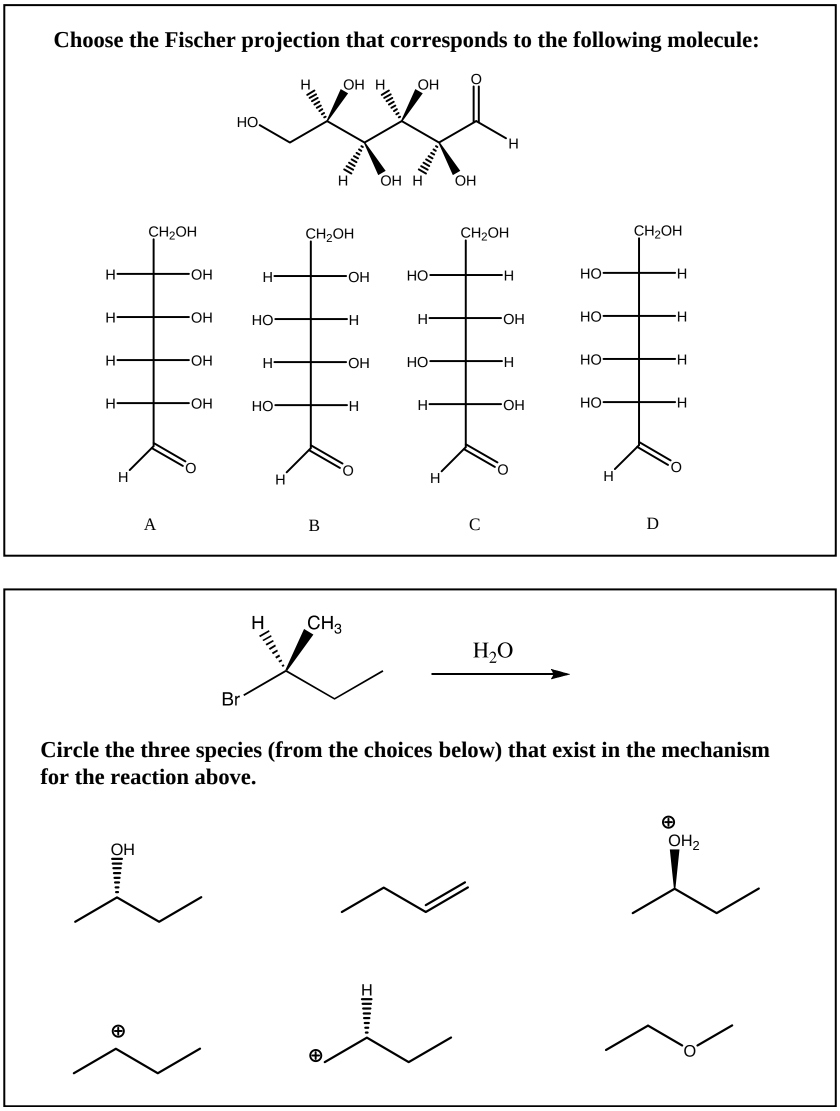

Figure 1. Examples of two problems from the test. The upper panel involves relating different diagrammatic representations and the lower one involves analysing the products of reactions.

this class. Imagistic training emphasised learning to mentally visualise molecular structures with the use of three-dimensional molecular models. Analytic training emphasised applying learned algorithms to transform molecular diagrams and rule-based reasoning while discouraging mental visualisation. Combined training gave equal emphasis to analytical and imagistic strategies.

Coding

Each strategy choice was coded a priori as imagistic, diagrammatic, spatial-analytic, or algorithmic (see examples in Table 1). Participants could also indicate if they guessed or did not know how they derived the answer; these answer choices were not included in the analysis. Write-in strategies (less than 2% of all reported strategies) were classified by two independent coders. Inter-rater agreement was 81% and discrepancies were resolved by consensus of the two coders.

Pearson correlations between the three spatial ability measures ranged from .40 to .54 ($p < .001$ in all cases). We created a composite spatial ability measure by converting students' scores to standardised Z-scores and taking the average of these scores. Students were classified as either higher or lower in spatial ability based on a median split of this composite measure.

TABLE 2
Descriptive statistics for number and proportion of each type of strategy reported before (pretest) and after (posttest) instruction

	Before instruction		*After instruction*	
	Number Mean (SD)	*Proportion Mean (SD)*	*Number Mean (SD)*	*Proportion Mean (SD)*
Strategy type				
Imagistic	2.59 (2.60)	0.50 (0.34)	5.40 (3.20)	0.37 (0.21)
Diagrammatic	1.20 (1.71)	0.20 (0.26)	3.88 (2.58)	0.24 (0.14)
Spatial analytic	0.24 (0.60)	0.04 (0.10)	3.02 (2.22)	0.20 (0.14)
Algorithmic	0.79 (1.03)	0.14 (0.20)	2.82 (1.45)	0.19 (0.09)
Total (all strategies)	4.83 (4.22)		15.11 (4.45)	

RESULTS

Cognitive change in problem-solving strategies

Table 2 reports both the number and the proportion of each type of strategy reported before and after instruction. Not surprisingly, students reported using more strategies after instruction than before, and this was true for all strategy types, $t(468) > 16$, $p < .001$, $d > 0.9$, in all cases. A comparison of the proportion of each type of strategy reported before and after instruction reveals that the proportion of imagistic strategies decreased significantly from before to after instruction, $t(468) = -7.42$, $p < .001$, $d = -0.46$. In contrast, the proportion of diagrammatic strategies, $t(468) = 3.16$, $p = .002$, $d = 0.19$, spatial analytic strategies, $t(468) = 20.94$, $p < .001$, $d = 1.32$, and algorithmic strategies, $t(468) = 4.38$, $p < .001$, $d = 0.32$, all increased. Thus, as expected, students depended largely on imagistic strategies at the beginning of instruction but switched to using a greater proportion of analytic methods by the end.

To examine how instructional method and spatial abilities affected the cognitive change from imagistic to analytic strategies, we examined the number of reported imagistic, diagrammatic, spatial analytic, and algorithmic strategies adopted after instruction in 3 (type of instruction: imagistic, analytic, or combined) × 2 (higher/lower spatial ability) analyses of covariance. Because there was some variability between groups in the strategies adopted at pretest,[1] the number of pretest strategies of each type was entered as a covariate in each analysis. Relevant descriptive statistics are shown in Figure 2.

Figure 2a shows the number of imagistic strategies reported by students with higher and lower spatial ability as a function of instruction. The analysis of covariance revealed a significant effect of instruction type, $F(2, 428) = 255.56$, $p = .002$, $\eta_p^2 = .03$. Pairwise comparisons revealed that students who had received combined (imagistic plus analytic) instruction reported more imagistic strategies ($M = 6.15$, $SD = 3.03$) than those who received either imagistic ($M = 5.23$, $SD = 3.02$, $d = 0.30$) or analytic instruction alone ($M = 4.92$, $SD = 3.06$, $d = 0.41$).[2] There was also a main effect of spatial ability, $F(1, 428) = 22.914$, $p < .001$, $\eta_p^2 = .05$, such that higher spatial individuals reported using more imagistic strategies ($M = 6.13$, $SD = 3.05$) than lower spatial individuals ($M = 4.74$, $SD = 3.01$, $d = 0.46$). The interaction of instruction and spatial ability was not significant ($F < 1$). In sum, students reported more imagistic strategies at the end of the semester if their instruction emphasised the combination of imagistic and analytic strategies and if they had higher spatial ability.

As shown in Figure 2b, there was also an effect of instruction on use of diagrammatic strategies, $F(2, 428) = 5.97$, $p = .003$, $\eta_p^2 = .03$. In pairwise comparisons, students reported more diagrammatic strategies when they received analytic ($M = 4.31$, $SD = 2.53$, $d = 0.38$) or mixed strategy instruction ($M = 4.15$, $SD = 2.54$, $d = 0.32$) than if they received instruction that emphasised imagistic strategies ($M = 3.34$, $SD = 2.53$). Interestingly, use of diagrammatic strategies was unrelated to spatial ability ($F < 1$) or the interaction of spatial ability and instruction ($F < 1$).

Figure 2c illustrates there was a main effect of instruction on report of spatial-analytic strategies after instruction, $F(2, 428) = 8.49$, $p < .001$, $\eta_p^2 = .04$. Pairwise comparisons indicated that students who received analytic instruction

[1] There was a significant difference only in the case of imagistic problem solving.

[2] The alpha level for all pairwise comparisons reported in this paper was .05, after Bonferroni adjustment for multiple comparisons.

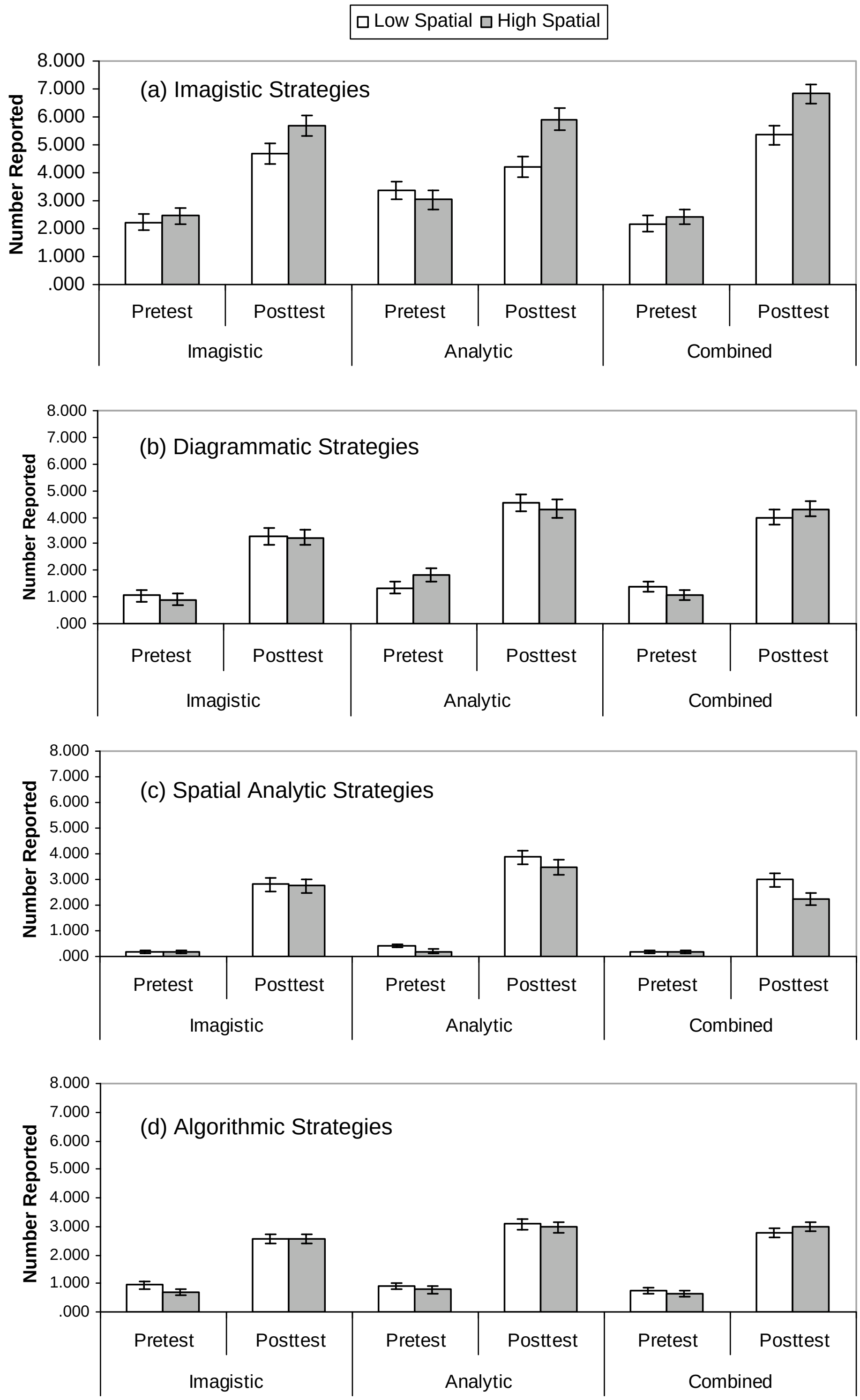

Figure 2. Mean number of strategies reported before (pretest) and after (posttest) instruction as a function of type of instruction received (imagistic, analytic, or combined) and spatial ability. Error bars show standard errors of the mean.

reported more spatial-analytic strategies ($M = 3.67$, $SD = 2.17$) than those who received imagistic ($M = 2.78$, $SD = 2.16$, $d = 0.41$) or combined ($M = 2.62$, $SD = 2.16$, $d = 0.48$) instruction. There was also a trend for students with lower spatial ability to adopt more spatial-analytic strategies than higher spatial individuals, $F(1, 428) = 3.29$, $p = .07$, $\eta_p^2 = .01$. The interaction of instruction and spatial ability was not significant ($F < 1$).

Finally, Figure 2d shows that instruction affected the adoption of algorithmic strategies after instruction, $F(2, 428) = 4.18$, $p = .016$, $\eta^2_p = .02$. Pairwise comparisons indicated that students who received analytic instruction used these strategies more often ($M = 3.00$, $SD = 1.39$) than those who received imagistic instruction ($M = 2.55$, $SD = 1.34$, $d = 0.33$). Students who received combined instruction ($M = 2.91$, $SD = 1.40$) did not differ significantly from the other two groups. Neither spatial ability nor the interaction of spatial ability and instruction had significant effects on use of algorithmic strategies ($F < 1$, in both cases).

Problem-solving performance

Not surprisingly, students solved more problems correctly at the end of the semester (6.74, $SD = 2.27$) than at the beginning (2.87, $SD = 1.38$), $t(468) = 33.33$, $p < .001$, $d = 2.06$, but even at the end of the semester, solution rate was well below the maximum possible score of 12, indicating that the problems were challenging. Students who reported using more strategies after instruction had better problem-solving performance, Pearson $r = .287$, $p < .001$. Specifically, reporting more diagrammatic, $r = .257$, $p < .001$, spatial analytic, $r = .115$, $p < .05$, and algorithmic strategies, $r = .186$, $p < .001$, was associated with greater problem-solving success. In contrast, the number of imagistic strategies reported at the end of the semester was unrelated to problem-solving success, $r = .029$, $p = .53$.

DISCUSSION

We observed cognitive change in students' reasoning strategies, which reflects a change in their mental models of molecular structures and processes, as they gained more knowledge in the domain of organic chemistry. At the beginning of an introductory course, half of the strategies reported by students involved constructing imagistic mental models of the molecules and mentally transforming these models using analogue processes such as mental rotation and perspective taking. By the end of the course, such imagistic strategies constituted about a third of the strategies reported, indicating a switch from more analogue mental models to more abstract representations. These results are consistent with research in other domains suggesting that imagistic strategies are associated with novelty, and as students gain more experience in a domain, they adopt more domain-specific analytic strategies (Lowrie & Kay, 2001; Schwartz & Black, 1996; Stieff, 2007).

The change from imagistic to analytic thinking was influenced by instruction. Specifically, students who received imagistic instruction reported fewer diagrammatic, spatial-analytic, and algorithmic strategies than the other groups, whereas those who receive analytic instruction reported more of these strategies. In contrast, students who received both imagistic and analytic instruction were more likely to use imagistic strategies at the end of the semester. Interestingly, students who were taught only imagistic strategies did not show this trend. This finding is consistent with research with third-grade mathematics students who adopt strategies selectively as a function of instruction (Heinze, Marschick, & Lipowsky, 2009).

Importantly, instruction did not determine students' mental models independent from other factors. The use of all types of strategies increased for all instruction groups. For example, students who received analytic instruction increased their use of imagistic strategies and students who received imagistic instruction increased their use of diagrammatic, spatial analytic, and algorithmic strategies. Moreover, ability influenced students' strategies such that those with good spatial abilities used relatively more imagistic strategies and tended to use few spatial-analytic strategies after instruction, compared to the other groups. Higher spatial ability individuals may prefer imagistic strategies because these strategies are not effortful for this group. In contrast, students with lower spatial ability may be more likely to adopt alternative analytic strategies because imagistic strategies are effortful for them. Thus, strategy choice appears to be adaptive. Spatial ability did not affect adoption of diagrammatic strategies, suggesting that this ability is more related to use of analogue imagery processes that operate on imagistic mental models than to strategies that involve modification of external visual-spatial representations. There was also no effect of spatial ability on the use of algorithmic strategies

Finally, this study supports the view that mental models of physical systems involve interplay between a range of representations and strategies from more imagistic to more abstract (Hegarty, 2004). A similar conclusion has been reached in studies of verbal reasoning tasks, such as linear syllogisms (e.g., Goel, 2007; Sternberg, 1980), and, more generally, the idea of multiple reasoning strategies is compatible with theories of dual processes in reasoning (Sloman, 1996). At the end of instruction, students who used a greater range of strategies, and adopted more diagrammatic, spatial analytic, and algorithmic strategies, were more successful problem solvers. In sum, successful problem solving in organic chemistry involves flexible strategy choice among a range of strategies. Research in other domains has also demonstrated higher success rates among problem solvers who employ a range of strategies across multiple problems compared to students who steadfastly employ the same strategies (Crowley & Siegler, 1993).

A possible limitation of our study is that we observed strategy choice during the first week of the organic chemistry class, when students could not be expected to have all the knowledge necessary to solve these problems. However, even at this early stage, students attempted some of these problems, and their preliminary attempts were primarily imagistic. Furthermore, when students report their strategies on organic chemistry problems immediately after learning the relevant content knowledge, they also reported primarily imagistic strategies that involve imagining rigid transformations of objects (Stieff et al., 2012). Initial mental models and inferences in chemistry may rely on analogies from our everyday experiences with rigid objects and from the common use of rigid ball-and-stick models of molecules in the domain. Our research suggests that, with experience, students add more abstract rules and heuristics to their available strategies for problem solving, but that the most successful problem solvers are those who integrate these domain-specific strategies with their naïve mechanistic models.

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